Principles of Macroeconometric Modeling

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INTRODUCTION TO THE SERIES

The aim of the series is to cover topics in economics, mathematical economics and econometrics, at a level suitable for graduate students or final year undergraduates specializing in economics. There is at any time much material that has become well established in journal papers and discussion series which still awaits a clear, self-contained treatment that can easily be mastered by students without considerable preparation or extra reading. Leading specialists will be invited to contribute volumes to fill such gaps. Primary emphasis will be placed on clarity, comprehensive coverage of sensibly defined areas, and insight into fundamentals, but original ideas will not be excluded. Certain volumes will therefore add to existing knowledge, while others will serve as a means of communicating both known and new ideas in a way that will inspire and attract students not already familiar with the subject matter concerned.

The Editors
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Preface

Two important new developments have occurred that have significant impact on the evolution of econometrics, namely, the end of the Cold War and the emergence of the information revolution in nearly all economies of the world. These major events require reconsideration and redrafting of some of the materials of the original edition.\footnote{Cf. L.R. Klein, Lectures in Econometrics with a chapter on Modelling Socialist Economy by W. Welfe, North-Holland, Amsterdam 1983.}

Keeping in mind that the first edition was not intended to cover all aspects of econometrics, but only the specification of the broad general structure of dynamic models and their application to problems of economic analysis, we set out in planning this new edition to re-draft those chapters or parts of chapters that are most sensitive to the two developments mentioned above.

In the original edition, there was a chapter by Władysław Welfe on models of the socialist economy, focusing on Poland. Most socialist economies have ceased to be planned economies and have either adapted to market mechanisms or have largely dismantled the socialist aspects. This is the case for Poland, which has become an economy in \textit{transition}—from plan to market. Professor Welfe shows in the present edition how the macroeconomic treatment of the Polish economy changes as this transition process is taking place. His treatment of this subject is applicable to many other economies in transition, especially those in Eastern Europe who were formerly members of the Council on Mutual Economic Assistance (CMEA). It is also relevant for some of the republics of the former Soviet Union and of Yugoslavia.

The information revolution has had significant effect on data flows, making them much more timely, accessible, and descriptive of more parts of the economy. At the same time, it has changed the industrial structure of many economies, giving rise to increasing importance of the tertiary sectors (services, e.g.). The new generations of hardware and software enable econometricians to handle larger and more complex problems, especially those that are data intensive and computer intensive.
The chapter on forecasting in this edition is affected by this development, and description of high-frequency forecasting from the data flow of monthly, weekly, daily statistical series has become much more prominent. A particular high-frequency system is described and analyzed in the chapter on forecasting.

Each chapter was reconsidered for changes, and some attention is devoted to a discussion of new fashions in econometric methodology that are presently very popular in macroeconometrics, namely time series analysis that is only loosely and sparsely related to structural economic analysis. The focus of this volume remains on the use of structural models that are closely tied to detailed social accounting systems. In many respects, the new methodologies in econometrics are largely based on well known approaches that have previously been considered in the development of econometrics or are extensions of structural models that either pay little attention to economic theory if the models are large, or are so aggregative that they are mainly useful as pedagogical models and not as working models of the entire economy. These considerations guide the revisions of chapters of the first edition that deal with dynamic analysis of macroeconometric systems.

The present volume retains the original structure of “Lectures . . .” and takes up principles of constructing dynamic macroeconometric models and their use in economic analyses and forecasting, while introducing many updates, revisions and extensions. The description of the econometric methodology has been limited to specific applications of time series analysis, and the title has been changed to “Principles of Macroeconometric Modeling”.

The first four chapters discuss the principles of specifying equations of structural macromodels, covering both developed market economies, transition economies and world-wide models. The remaining chapters cover some major issues in the use of macromodels. The point of departure is model simulation, especially of the prevailing non-linear models, which is followed by model validation. The analysis of model dynamics covers economic fluctuations and the relevant implications of non-stationarity. The use of macromodels in policy analysis is presented next; it includes multiplier analysis and scenario simulations. The monograph ends up with forecasting being a special case of simulation analysis.

Aleksander Welfe cooperated closely in making numerous revisions and extensions, especially in the second part of the monograph. He was also responsible for the majority of editorial issues.

L.R. Klein
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Chapter 1

Models of the Economy as a Whole

What is a model?

A schematic simplification that strips away the nonessential aspects to reveal the inner workings, shape, or design of a more complicated mechanism constitutes a model. Social systems are enormously complicated, so much so that we can never grasp the complete explanation of all aspects of society at once. We break the problem into parts, but even that is not sufficient for complete human comprehension; therefore, modeling is an important step. A social model consists of simplifying assumptions; approximate but understandable relationships; and some explanation of reality. It, by itself, is not reality, but merely a simplified picture of reality that man can understand.

Models are far reaching in social analysis. In this volume, we are going to concentrate on economic models because we feel completely at ease in that area, but inevitably it will become necessary to break outside the confines of the purely economic aspects of behavior to more general social models. The necessity will arise because economics alone is not all-embracing or self-contained. To explain some basic aspects of economic behavior, it will be necessary to explain related aspects of more general social behavior, encompassing politics, demography, and even social engineering.
At the beginning, the discussion will be entirely concerned with quantitative models that are capable of mathematical expression. Later, however, more general possibilities will be considered—historical, qualitative, non-mathematical models.

**A macroeconomic model — A new approach**

The theory of employment and output determination developed in the 1930s formed the basis of abstract and, later, statistical model building for the economy as a whole. The real meaning of the *Keynesian Revolution* become clear when model comparison of alternative systems—the classical, neoclassical, Keynesian, Marxian—were formulated side-by-side in mathematical equation systems. Model building took an unfortunate doctrinal turn at that time. Although we participated as fully as anyone else in that approach to model building, we have lately come to feel that a more rewarding approach that is neutral as far as doctrine is concerned will be through the accounting structure.

Model structure is both art and science. Inspiration, leads, pieces of evidence, and many sources of information are needed to put together a good model. If all the economic accounts associated with a problem are laid out in advance with the necessary accounting balance equations stated, we can see immediately what is needed in order to get a full explanation of the whole system. As an instructive example of a closed accounting system and an associated model, let us consider a simplified national income accounting (NIA) (see Table 1) system.¹

The units of economic action:
- Households \((H)\)
- Businesses \((B)\)
- Government \((G)\)
- Foreigners \((F)\)

The accounts: Balance systems of receipts and expenditures, on a double entry basis. These are income accounts for each unit.

¹There is a good deal of generality in this and succeeding accounting designs in this chapter. The same is true of the associated macromodels, yet this entire exposition is pitched for the structure of the typical OECD (Western industrial market) economy. A different structure would be appropriate for the centrally planned, transition and developing economies.
Table 1
The NIA system

<table>
<thead>
<tr>
<th>Households (H)</th>
<th>Business (B)</th>
<th>Government (G)</th>
<th>Foreigners (F)</th>
<th>Sources and Uses (S/U)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(EH1) Consumer expenditures $= (RB1)$</td>
<td>(RH1) Receipts from $B = (EB1)$</td>
<td>(RG1) Taxes received from $H = (EH2)$</td>
<td>(RF1) Imports $= (EB3)$</td>
<td>(S1) Personal saving $= (EH3)$</td>
</tr>
<tr>
<td>(EH2) Taxes paid by $H = (RG1)$</td>
<td>(RH2) Receipts from $G = (EG1)$</td>
<td>(RG2) Taxes received from $B = (EB2)$</td>
<td>(EF2) Foreign saving $= (S4)$</td>
<td>(S2) Business saving $= (EB5)$</td>
</tr>
<tr>
<td>(EH3) Personal Saving $= (S1)$</td>
<td></td>
<td>(EG1) Wages and transfers paid to $H = (RH2)$</td>
<td></td>
<td>(S3) Government saving $= (EG3)$</td>
</tr>
<tr>
<td>(EB1) Income paid to $H$</td>
<td>(RB1) Consumer expenditures $= (EH1)$</td>
<td></td>
<td></td>
<td>(S4) Foreign saving $= (EF2)$</td>
</tr>
<tr>
<td>(EB2) Taxes paid by $B = (RG2)$</td>
<td>(RB2) Sales to $G = (EG2)$</td>
<td></td>
<td></td>
<td>(S5) Capital consumption $= (EB4)$</td>
</tr>
<tr>
<td>(EB3) Imports $= (RF1)$</td>
<td>(RB3) Exports $= (EF1)$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(EB4) Capital consumption $= (S5)$</td>
<td>(RB4) Sales by $B$ to $B$, Investment $= (U1)$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(EB5) Business saving $= (S2)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In order to make the system double entry, there must be provision for separate entries for balancing items in each account—the various savings accounts. These are offset in the final account for Sources and Uses (S/U) by entries for gross investment (final sales by business to other business) and depreciation or capital consumption.

In this system, every entry occurs twice and is labeled on the left by one source. On the right, we label the other place in the accounting system where it occurs. There are 28 entries and, by the dual principle, we need
only 14 equations to model the system. Associated with each account, there are balance equations, five in all:

\[(EH1) + (EH2) + (EH3) = (RH1) + (RH2), \quad (1.1)\]

\[(EB1) + (EB2) + (EB3) + (EB4) + (EB5) = (RB1) + (RB2) + (RB3) + (RB4), \quad (1.2)\]

\[(EG1) + (EG2) + (EG3) = (RG1) + (RG2), \quad (1.3)\]

\[(EF1) + (EF2) = (RF1), \quad (1.4)\]

\[(U1) = (S1) + (S2) + (S3) + (S4) + (S5). \quad (1.5)\]

Next, we must formulate 9 more statistical equations for the behavioral, institutional, and technological structure of the economy. Some plausible relationships are:

consumption function

\[(EH1) = \alpha_0 + \alpha_1[(RH1) + (RH2) - (EH2)] + \alpha_2(EH1)_{-1} + \epsilon_6, \quad (1.6)\]

personal tax function

\[(EH2) = \beta_0 + \beta_1[(RH1) + (RH2)] + \epsilon_7, \quad (1.7)\]

wage equation

\[(EB1) = \gamma_0 + \gamma_1[(RB1) + (RB2) + (RB3) + (RB4)] + \epsilon_8. \quad (1.8)\]

business tax function

\[(EB2) = \delta_0 + \delta_1[(RB1) + (RB2) + (RB3) + (RB4) - (EB1) - (EB3) - (Eb4)] + \epsilon_9. \quad (1.9)\]

import function

\[(EB3) = \epsilon_0 + \epsilon_1[(RB1) + (RB2) + (RB3) + (RB4)] + \epsilon_{10}, \quad (1.10)\]
depreciation equation

\[(EB4) = \xi_1(RB4) + \xi_2(EB4)_{-1} + e_{11}, \quad (1.11)\]

investment function

\[(RB4) = \eta_0 + \eta_1[(RB1) + (RB2) + (RB3) + (RB4) - (EB3)]
+ \eta_2(RB4)_{-1} + e_{12}, \quad (1.12)\]

\[(EG1) + (EG2) = \text{exogenous}, \quad (1.13)\]

\[(EF1) = \text{exogenous}. \quad (1.14)\]

The first 5 equations of the model are self evident by virtue of the accounting balances used in data construction and presentation. The last 2 are not really structural equations; they are simply part of the specification of the model, explaining what is to be internally generated and what is to be imposed from the outside. The external or exogenous variables are driving forces in the system. Equation (1.13) states simply that total government expenditures (wage payments plus purchases) are not determined on the basis of rules, either behavioral or statutory; they are set by the legislative and administrative branches of government in a way that serves contemporary political and economic interests. In a more complicated system, parts of the government spending process can be explained endogenously, but that discussion is to be given later.

Equation (1.14) defines exports as exogenous, and this is a first approximation, assuming that exports depend primarily on external economic activity. As in the case of government expenditures, an endogenous treatment of exports is postponed until the development of more elaborate and complicated systems below.

Equations (1.6)–(1.12) are simple structural equations of the model. They are all expressed in linear form, although that is not necessary. It is done only for convenience, simplicity, and because linearity may provide a fairly good empirical approximation in the present context. They are all stochastic, containing additive error terms \(e_6, \ldots, e_{12}\). The probability properties of the errors are used in making statistical inferences from sample data about the sizes of the unknown coefficients in the linear expressions. The model is dynamic, by virtue of the time lags introduced in Equations (1.6), (1.11) and (1.12), associated with the processes of consumer spending, capi-
tal consumption, and capital formation. The lag variable, carried over from the previous observation period, has the same role as an exogenous variable in the first period of a solution of the model. Lags and exogenous variables are inputs to the solution process. As additional solution periods are developed, after the first, exogenous values must be repeatedly obtained as external input, while lagged values can generated from prior solutions as delayed internal input. In Chapter 5 on Simulation, dynamic solutions of models with lags are taken up in detail.

The accounting structure tells us what to look for. It gives little or no insight into the particular specification of structural equations. As an exercise and with that alone in mind, we might set for ourselves the task of devising an interrelated model made up entirely of interrelationships among the entries in the simple NIA accounting system.

Equation (1.6) is the familiar propensity to consume. It expresses personal consumption expenditures as a function of disposable personal income, i.e., income payments to persons plus transfers from the government less personal taxes paid to government. Previous consumption is introduced as an added explanatory variable to show either habit persistence or the transformation of a distributed lag of past disposable income values, stretching back in time with geometrically declining weights.

In constructing the concept of disposable income in (1.6), we subtracted personal tax payments from personal income. These taxes are not, however, independent of the size of personal income. By law they vary with income and a great many other special factors according to a whole schedule of income taxes. The tax laws apply, however, to individuals in a complicated nonlinear way. In the aggregate, there is a simple smooth relation between total taxes paid and income received, although the parameters of this function will have to be re-evaluated when tax laws change substantially. Also, large inflationary movements and other causes of income redistribution by tax bracket can affect the coefficients of a simple linear relation; therefore much care must be exercised to allow for parameter shifts in the simple linear form.

Transfer payments from government to persons could be treated like taxes, but with opposite direction of flow. This was not done in the present model in order to condense the accounts as much as possible. Transfer payments are lumped with other payments (wages and interest) from government to persons.

The wage Equation (1.8) shows the distribution of payments in the business account between wages and other factor payments, principally profits. In this system, business retained profits are entered as business saving.