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Student Resource Manual

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to accompany

Introduction to Statistical Quality Control

Fourth Edition

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PREFACE

The purpose of this Resource Manual is to provide the student with an in-depth understanding of how to apply the concepts presented in the *Introduction to Statistical Quality Control*, 4th ed., by Douglas C. Montgomery. Along with detailed instructions on how to solve the chapter exercises, insights from practical application are also shared.

In general, solutions have been provided for "Answers to Selected Exercises" listed at the end of the text. However, occasionally a group of "continued" exercises is presented and provide the student with a full solution for a specific data set.

The exercises were solved using statistical software applications widely available for the personal computer:

- Design-Expert® Version 6.0.3 from Stat-Ease, www.statease.com
- JMP® Version 3.2.6 from SAS Institute, www.JMPdiscovery.com
- Excel 2000 from Microsoft®, www.microsoft.com
- MINITAB™ Release 13.1, www.minitab.com

Files with the relevant data sets for each chapter are available for download at www.wiley.com/college/montgomery, under the Online Student Resources for the SQC text.

CONTENTS

CHAPTER 1

Quality Improvement in the Modern Business Environment

CHAPTER GOALS

After completing this chapter, you will be able to:

1. Define quality improvement and the role of statistical methods in variability reduction.

The modern definition of quality, "**Quality** is inversely proportional to variability" (text p. 4), implies that product quality increases as variability in important product characteristics decreases. **Quality improvement** can then be defined as "... the reduction of variability in processes and products" (text p. 6). Since the early 1900's, statistical methods have been used to control and improve quality. In the *Introduction to Statistical Quality Control*, 4th ed., by Douglas C. Montgomery, methods applicable in the key areas of *process control*, *design of experiments*, and *acceptance sampling* are presented.

To understand the potential for application of statistical methods, it may help to envision the system that creates a product as a "black box" (text Figure 1-3). The output of this black box is a product whose quality is defined by one or more quality characteristics that represent dimensions such as conformance to standards, performance, or reliability. Product quality can be evaluated with *acceptance sampling plans*. These plans are typically applied to either the output of a process or the input raw materials and components used in manufacturing. Application of process control techniques (such as control charts) or statistically designed experiments can achieve significant reduction in variability.

Black box inputs are categorized as "incoming raw materials and parts," "controllable inputs," and "uncontrollable inputs."

The quality of incoming raw materials and parts is often assessed with *acceptance sampling plans*. As material is received from suppliers, incoming lots are inspected then dispositioned as acceptable or unacceptable. Once a history of high quality material is established, a customer may accept the supplier's *process control* data in lieu of incoming inspection results.

"Controllable" and "uncontrollable" inputs apply to incoming materials, process variables, and environmental factors. For example, it may be difficult to control the temperature in a heat-treating oven in the sense that some areas of the oven may be cooler while some areas may be warmer. Properties of incoming materials may be very difficult to control. For example, the moisture content and proportion of hardwood in trees used for papermaking have a significant impact on the quality characteristics of the finished paper. Environmental variables such as temperature and relative humidity are often hard to control precisely.

Whether or not controllable and uncontrollable inputs are significant can be determined through process characterization. *Statistically designed experiments* are extremely helpful in characterizing processes and optimizing the relationship between incoming materials, process variables, and product characteristics

Although the initial tendency is to think of manufacturing processes and products, the statistical methods presented in this text can also be applied to *business* processes and products, such as financial transactions and services. In some organizations the opportunity to improve quality in these areas is even greater than it is in manufacturing.

Various quality philosophies and management systems are briefly described in the text; a common thread is the necessity for continuous improvement to increase productivity and reduce cost. The technical tools described in the text are essential for successful quality improvement. Quality management systems alone do not reduce variability and improve quality.

CHAPTER 2

Modeling Process Quality

CHAPTER GOALS

After completing this chapter, you will be able to:

1. Describe variation using the following graphical methods: Stem and leaf plot; Histogram or frequency distribution; Box Plot; Probability Distributions
2. Describe data using the following numerical estimates: Average; Variance; Standard Deviation
3. Understand the difference between continuous and discrete distributions
4. Recognize and use the following discrete distributions: Hypergeometric; Binomial; Poisson; Pascal
5. Recognize and use the following continuous distributions: Normal; Exponential; Gamma; Weibull
6. Apply the Central Limit Theorem
7. Use approximation methods for probability distributions

Exercises

2-1. The fill volume of a soft-drink beverage is being analyzed for variability. Ten bottles, randomly selected from the process, are measured, and the results are as follows (in fluid ounces): 10.05, 10.03, 10.02, 10.04, 10.05, 10.01, 10.02, 10.02, 10.03, 10.01.

(a) Calculate the sample average.

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{10.05 + 10.03 + \cdots + 10.01}{10} = 10.028 \text{ oz}$$

(b) Calculate the sample standard deviation.

$$S = \sqrt{\frac{\sum_{i=1}^{n} x_i^2 - \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n}}{n-1}} = \sqrt{\frac{(10.05^2 + \cdots + 10.01^2) - \frac{(10.05 + \cdots + 10.01)^2}{10}}{10-1}} = 0.015 \text{ oz}$$

The Minitab basic statistics output for this data set is:

Variable	N	Mean	Median	TrMean	StDev	SE Mean
Ex2-1	10	10.028	10.025	10.028	0.015	0.005
Variable	Minimum	Maximum	Q1	Q3		
Ex2-1	10.010	10.050	10.018	10.043		

2-3. The nine measurements that follow are furnace temperatures recorded on successive batches in a semiconductor manufacturing process (units are F):

953	955	948
951	957	949
954	950	959

(a) Calculate the sample average.

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{953 + 951 + \cdots + 959}{9} = 952.9 \text{ °F}$$

(b) Calculate the sample standard deviation.

$$S = \sqrt{\frac{\sum_{i=1}^{n} x_i^2 - \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n}}{n-1}} = \sqrt{\frac{(953^2 + \cdots + 959^2) - \frac{(953 + \cdots + 959)^2}{9}}{9-1}} = 3.7 \text{ °F}$$

The Minitab basic statistics output for this data set is:

Variable	N	Mean	Median	TrMean	StDev	SE Mean
Ex2-3	9	952.89	953.00	952.89	3.72	1.24
Variable	Minimum	Maximum	Q1	Q3		
Ex2-3	948.00	959.00	949.50	956.00		

2-5. Yield strengths of circular tubes with end caps are measured. The first yields (in kN) are as follows:

96	102	104	108
126	128	150	156

(a) Calculate the sample average.

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{96 + 102 + \cdots + 156}{8} = 121.25 kN$$

(b) Calculate the sample standard deviation.

$$S = \sqrt{\frac{\sum_{i=1}^{n} x_i^2 - \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n}}{n-1}} = \sqrt{\frac{(96^2 + \cdots + 156^2) - \frac{(96 + \cdots + 156)^2}{8}}{8-1}} = 22.63 kN$$

The Minitab basic statistics output for this data set is:

Variable	N	Mean	Median	TrMean	StDev	SE Mean
Ex2-5	8	121.25	117.00	121.25	22.63	8.00
Variable	Minimum	Maximum	Q1	Q3		
Ex2-5	96.00	156.00	102.50	144.50		

2-11. Consider the chemical process yield data in Exercise 2-7. Calculate the sample average and standard deviation.

Exercise 2-7 Data

94.1	87.3	94.1	92.4	84.6	84.4
93.2	84.1	92.1	90.6	83.6	86.6
90.6	90.1	96.4	89.1	85.4	91.7
91.4	95.2	88.2	88.8	89.7	87.5
88.2	86.1	86.4	86.4	87.6	84.2
86.1	94.3	85.0	85.1	85.1	85.1
95.1	93.2	84.9	84.0	89.6	90.5
90.0	86.7	87.3	93.7	90.0	95.6
92.4	83.0	89.6	87.7	90.1	88.3
87.3	95.3	90.3	90.6	94.3	84.1
86.6	94.1	93.1	89.4	97.3	83.7
91.2	97.8	94.6	88.6	96.8	82.9
86.1	93.1	96.3	84.1	94.4	87.3
90.4	86.4	94.7	82.6	96.1	86.4
89.1	87.6	91.1	83.1	98.0	84.5

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{94.1+93.2+\cdots+84.5}{90} = 89.476$$

$$S = \sqrt{\frac{\sum_{i=1}^{n} x_i^2 - \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n}}{n-1}} = \sqrt{\frac{(94.1^2+\cdots+84.5^2) - \frac{(94.1+\cdots+84.5)^2}{90}}{90-1}} = 4.158$$

The Minitab basic statistics output for this data set is:

```
Variable           N       Mean     Median     TrMean      StDev    SE Mean
Ex2-7             90     89.476     89.250     89.391      4.158      0.438
Variable     Minimum    Maximum         Q1         Q3
Ex2-7         82.600     98.000     86.100     93.125
```

2-15. Suppose that two fair dice are tossed and the random variable observed - say, x -is the sum of the two up faces. Describe the sample space of this experiment, and determine the probability distribution of x.

x: {the sum of two up dice faces}
sample space: {2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}
Calculate the probability of rolling a 2, obtained by rolling a 1 on each die:

$$\Pr\{x=2\} = \Pr\{1,1\} = \frac{1}{6}\times\frac{1}{6} = \frac{1}{36}$$

Calculate the probability of rolling a 3, 1 and 2 or 2 and 1:

$$\Pr\{x=3\} = \Pr\{1,2\} + \Pr\{2,1\} = \frac{1}{6}\times\frac{1}{6} + \frac{1}{6}\times\frac{1}{6} = \frac{2}{36}$$

Calculate the probability of rolling a 4:

$$\Pr\{x=4\} = \Pr\{1,3\} + \Pr\{2,2\} + \Pr\{3,1\} = \frac{1}{6}\times\frac{1}{6} + \frac{1}{6}\times\frac{1}{6} + \frac{1}{6}\times\frac{1}{6} = \frac{3}{36}$$

The entire sample space is defined as:

$$p(x) = \begin{cases} 1/36; x=2 \\ 2/36; x=3 \\ 3/36; x=4 \\ 4/36; x=5 \\ 5/36; x=6 \\ 6/36; x=7 \\ 5/36; x=8 \\ 4/36; x=9 \\ 3/36; x=10 \\ 2/36; x=11 \\ 1/36; x=12 \\ 0;\ \text{otherwise} \end{cases}$$

2-17. A mechatronic assembly is subjected to a final functional test. Suppose that defects occur at random in these assemblies, and that defects occur according to a Poisson distribution with parameter $\lambda = 0.02$.

(a) What is the probability that an assembly will have exactly one defect?

This is a Poisson distribution with parameter $\lambda = 0.02$, $x \sim \text{POI}(0.02)$.

$$\Pr\{x=1\} = p(1) = \frac{e^{-0.02}(0.02)^1}{1!} = 0.0196$$

(b) What is the probability that an assembly will have one or more defects?

$$\Pr\{x \geq 1\} = 1 - \Pr\{x=0\} = 1 - p(0) = 1 - \frac{e^{-0.02}(0.02)^0}{0!} = 1 - 0.9802 = 0.0198$$

(c) Suppose that you improve the process so that the occurrence rate is cut in half to $\lambda = 0.01$. What effect does this have on the probability that an assembly will have one or more defects?

This is a Poisson distribution with parameter $\lambda = 0.01$, $x \sim \text{POI}(0.01)$.

$$\Pr\{x \geq 1\} = 1 - \Pr\{x=0\} = 1 - p(0) = 1 - \frac{e^{-0.01}(0.01)^0}{0!} = 1 - 0.9900 = 0.0100$$

Cutting the rate at which defects occur reduces the probability of one or more defects approximately half, from 0.0198 to 0.0100.

2-19. The random variable x takes on the values 1, 2, or 3 with probabilities $(1+3k)/3$, $(1+2k)/3$, and $(0.5+5k)/3$ respectively.

(a) Find the appropriate value of k.

$$p(x)=\begin{cases}(1+3k)/3; & x=1\\(1+2k)/3; & x=2\\(0.5+5k)/3; & x=3\\0; & \text{otherwise}\end{cases}$$

To solve for k, use $F(x)=\sum_{i=1}^{\infty}p(x_i)=1$

$$\frac{(1+3k)+(1+2k)+(0.5+5k)}{3}=1$$

$$\frac{2.5+10k}{3}=1$$

$$10k=0.5$$

$$k=0.05$$

(b) Find the mean and variance of x.

$$\mu=\sum_{i=1}^{3}x_ip(x_i)=1\times\left[\frac{1+3(0.05)}{3}\right]+2\times\left[\frac{1+2(0.05)}{3}\right]+3\times\left[\frac{0.5+5(0.05)}{3}\right]=1.867$$

$$\sigma^2=\sum_{i=1}^{3}x_i^2p(x_i)-\mu^2=1^2(0.383)+2^2(0.367)+3^2(0.250)-1.867^2=0.615$$

(c) Find the cumulative distribution function.

Apply $k=0.05$ to earlier equations-*:

$$F(x)=\begin{cases}\frac{1.15}{3}=0.383; x=1\\\frac{1.15+1.1}{3}=0.750; x=2\\\frac{1.15+1.1+0.75}{3}=1.000; x=3\end{cases}$$

2-21. A manufacturer of electronic calculators offers a 1-year warranty. If the calculator fails for any reason during this period, it is replaced. The time to failure is well modeled by the following probability distribution:

$$f(x)=0.125e^{-0.125x} \qquad x>0$$

(a) What percentage of the calculators will fail within the warranty period?

This is an exponential distribution with parameter $\lambda=0.125$:

$\Pr\{x\le 1\}=F(1)=1-e^{-0.125(1)}=0.118$

Approximately 11.8% will fail during the first year.

(b) The manufacturing cost of a calculator is \$50, and the profit per sale is \$25. What is the effect of warranty replacement on profit?

Mfg. cost = \$50/calculator Sale profit = \$25/calculator

Net profit = \$[-50(1 + 0.118) + 75]/calculator = \$19.10/calculator.

The replacement warranty effect decreases profit by 25 – 19.10 = \$5.90/calculator.

2-23. A production process operates with 2% nonconforming output. Every hour a sample of 50 units of product is taken, and the number of nonconforming units is counted. If one or more nonconforming units are found, the process is stopped and the quality control technician must search for the cause of nonconforming production. Evaluate the performance of this decision rule.

This is a binomial distribution with parameter $p=0.02$ and $n=50$. The process is stopped if $x\ge 1$.

$$\Pr\{x\ge 1\}=1-\Pr\{x<1\}=1-\Pr\{x=0\}=1-\binom{50}{0}(0.02)^0(1-0.02)^{50}=1-0.364=0.636$$

This decision rule means that 63.6% of the samples will have one or more nonconforming units, and the process will be stopped to look for a cause. This is a somewhat difficult operating situation.

2-25. A random sample of 100 units is drawn from a production process every half hour. The fraction of nonconforming product manufactured is 0.03. What is the probability that $\hat{p}\le 0.04$ if the fraction nonconforming really is 0.03?

This is a binomial distribution with parameter $p=0.03$ and $n=100$.

$$\Pr\{\hat{p}\le 0.04\}=\Pr\{x\le 4\}=\sum_{x=0}^{4}\binom{100}{x}(0.03)^x(1-0.03)^{(100-x)}$$

$$=\binom{100}{0}(0.03)^0(1-0.03)^{100}+\binom{100}{1}(0.03)^1(1-0.03)^{99}+\cdots+\binom{100}{4}(0.03)^4(1-0.03)^{96}=0.818$$

Therefore, the probability is 0.818 that sample fraction nonconforming can be $\hat{p}\le 0.04$ if the population fraction nonconforming really is $p=0.03$.

2-27. An electronic component for a laser range-finder is produced in lots of size N=25. An acceptance testing procedure is used by the purchaser to protect against lots that contain too many nonconforming components. The procedure consists of selecting five components at random from the lot (without replacement) and testing them. If none of the components is nonconforming, the lot is accepted.

(a) If the lot contains three nonconforming components, what is the probability of acceptance?

This is a hypergeometric distribution with $N = 25$ and $n = 5$, without replacement.

$$\Pr\{\text{acceptance}\} = p(x) = \frac{\binom{D}{x}\binom{N-D}{n-x}}{\binom{N}{n}}$$

Given $D = 3$ and $x = 0$:

$$\Pr\{\text{acceptance}\} = p(0) = \frac{\binom{3}{0}\binom{25-3}{5-0}}{\binom{25}{5}} = \frac{(1)(26,334)}{(53,130)} = 0.496$$

(b) Calculate the desired probability in (a) using the binomial approximation. Is this approximation satisfactory? Why or why not?

For the binomial approximation to the hypergeometric,

$$\Pr\{\text{acceptance}\} = p(x) = \binom{n}{x}(p)^x(1-p)^{n-x}$$

$p = \frac{D}{N} = \frac{3}{25} = 0.120$ and $n = 5$.

$$\Pr\{\text{acceptance}\} = p(0) = \binom{5}{0}(0.120)^0(1-0.120)^5 = (1)(1)(0.528) = 0.528$$

This approximation, though close to the exact for $x = 0$, violates the rule-of-thumb that $n/N = 5/25 = 0.20$ be less than the suggested 0.1. The binomial approximation is not satisfactory in this case.

(c) Suppose the lot size N=150. Would the binomial approximation be satisfactory in this case?

For $N = 150$, $n/N = 5/150 = 0.033 \le 0.1$, so the binomial approximation would be a satisfactory approximation the hypergeometric in this case.

(d) Suppose that the purchaser will reject the lot with the decision rule of finding one or more nonconforming components in a sample of size n, and wants the lot to be rejected with probability at least 0.95 if the lot contains five or more nonconforming components. How large should the sample size n be?

Find n to satisfy $\Pr\{x \ge 1 \mid D \ge 5\} \ge 0.95$, or equivalently $\Pr\{x = 0 \mid D = 5\} < 0.05$.

$$p(x) = \frac{\binom{D}{x}\binom{N-D}{n-x}}{\binom{N}{n}}$$

$$p(0) = \frac{\binom{5}{0}\binom{25-5}{n-0}}{\binom{25}{n}} = \frac{\binom{5}{0}\binom{20}{n}}{\binom{25}{n}}$$

try $n = 10$

$$p(0) = \frac{\binom{5}{0}\binom{20}{10}}{\binom{25}{10}} = \frac{(1)(184,156)}{(3,268,760)} = 0.057$$

try $n = 11$

$$p(0) = \frac{\binom{5}{0}\binom{20}{11}}{\binom{25}{11}} = \frac{(1)(167,960)}{(4,457,400)} = 0.038$$

Let sample size $n = 11$.

2-29. A textbook has 500 pages on which typographical errors could occur. Suppose that there are exactly 10 such errors randomly located on those pages. Find the probability that a random selection of 50 pages will contain no errors. Find the probability that 50 randomly selected pages will contain at least two errors.

This is a hypergeometric distribution with $N = 500$ pages, $n = 50$ pages, and $D = 10$ errors. Checking $n/N = 50/500 = 0.1 \le 0.1$, the binomial distribution can be used to approximate the hypergeometric, with $p = \frac{D}{N} = \frac{10}{500} = 0.020$.

$$\Pr\{\text{acceptance}\} = p(x) = \binom{n}{x}(p)^x(1-p)^{n-x}$$

$$\Pr\{x = 0\} = p(0) = \binom{50}{0}(0.020)^0(1-0.020)^{50-0} = (1)(1)(0.364) = 0.364$$

$$\Pr\{x \ge 2\} = 1 - \Pr\{x \le 1\} = 1 - [\Pr\{x = 0\} + \Pr\{x = 1\}] = 1 - p(0) - p(1)$$

$$= 1 - 0.364 - \binom{50}{1}(0.020)^1(1-0.020)^{50-1} = 1 - 0.364 - 0.372 = 0.264$$

2-31. Glass bottles are formed by pouring molten glass into a mold. The molten glass is prepared in a furnace lined with firebrick. As the firebrick wears, small pieces of brick are mixed into the molten glass and finally appear as defects (called "stones") in the bottle. If we can assume that stones occur randomly at the rate of 0.00001 per bottle, what is the probability that a bottle selected at random will contain at least one such defect?

This is a Poisson distribution with $\lambda = 0.00001$ stones/bottle. The Poisson distribution is $\Pr(x) = \dfrac{e^{-\lambda}\lambda^x}{x!}$.

$$\Pr\{x \geq 1\} = 1 - \Pr\{x = 0\} = 1 - \frac{e^{-0.00001}(0.00001)^0}{0!} = 1 - 0.99999 = 0.00001$$

2-33. A production process operates in one of two states: in the in-control state, in which most of the units produced conform to specifications, and an out-of-control state, in which most of the units produced are defective. The process will shift from the in-control to the out-of-control state at random. Every hour, a quality control technician checks the process, and if it is in the out-of-control state, the technician detects this with a probability p. Assume that when the process shifts out of control it does so immediately following a check by the inspector, and once a shift has occurred, the process cannot automatically correct itself. If t denotes the number of periods the process remains out of control following a shift before detection, find the probability distribution of t. Find the mean number of periods the process will remain in the out-of-control state.

The distribution for t is based on the number of periods for which the process shifted from in control to out of control, 1, with probability $(p)^1$, and the periods from which the process remains in the out of control state without a shift, $t - 1$, with probability $(1 - p)^{t-1}$. There is only one permutation for this situation. Hence, the distribution is the combination of the two states as follows:

$\Pr(t) = p(1-p)^{t-1}; \quad t = 1,2,3,\ldots$

The mean is calculated per equation 2-5b: $\mu = \sum_{i=1}^{\infty} x_i p(x_i)$ as follows:

$$\mu = \sum_{t=1}^{\infty} t\left[p(1-p)^{t-1}\right] = p\frac{d}{dq}\left[\sum_{t=1}^{\infty} q^t\right] = \frac{1}{p}$$

2-35. The tensile strength of a metal part is normally distributed with a mean of 40 lb and standard deviation 8 lb. If 50,000 parts are produced, how many would fail to meet a minimum specification of 34-lb tensile strength? How many would have a tensile strength in excel of 48 lb?

$x \sim N(40, 8^2); \quad n = 50{,}000$

How many fail the minimum specification, LSL = 34 lb.? Utilizing the standard normal distribution, Appendix II:

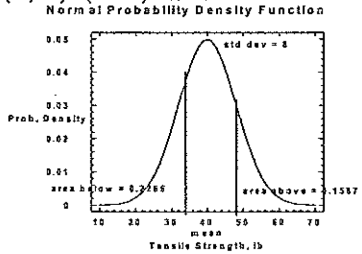
$$\Pr\{x \leq 34\} = \Pr\left\{z \leq \frac{34-40}{8}\right\} = \Pr\{z \leq -0.75\} = \Phi(-0.75) = 0.2266$$

Therefore, the number failing the minimum specification is (50,000) × (0.2266) = 11,330.

How many exceed 48 lb.? Again, utilizing the standard normal distribution, Appendix II:

$$\Pr\{x > 48\} = 1 - \Pr\{x \leq 48\} = 1 - \Pr\left\{z \leq \frac{48-40}{8}\right\} = 1 - \Pr\{z \leq 1.00\} = 1 - \Phi(1.00) = 1 - 0.8413 = 0.1587$$

The number that exceed 48 lb. is (50,000) × (0.1587) = 7935.



2-37. Continuation of Exercise 2-36. Reconsider the power supply manufacturing process in Exercise 2-36. Suppose we wanted to improve the process. Can shifting the means reduce the number of nonconforming units produced? How much would the process variability need to be reduced in order to have all but one of 1000 units conform to the specifications?

Exercise 2-36. The output voltage of a power supply is normally distributed with mean 12 V and standard deviation 0.05 V. If the lower and upper specifications for voltage are 11.90 V and 12.10 V, respectively, what is the probability that a power supply selected at random will conform to the specifications on voltage?

The process, with mean 12V, is currently centered between the specification limits (target = 12V). Shifting the process mean in either direction would increase the number of nonconformities produced. Desire Pr{conformance} = 1 / 1000 = 0.001. Assume that the process remains centered between the specification limits at 12V. Need $\Pr\{x \leq \text{LSL}\} = 0.001 / 2 = 0.0005$.

$\Phi(z) = 0.0005$

$z = \Phi^{-1}(0.0005) = -3.29$

$$z = \frac{\text{LSL} - \bar{x}}{s}, \quad \text{so } s = \frac{\text{LSL} - \bar{x}}{z} = \frac{11.90 - 12}{-3.29} = 0.03$$

Process variance must be reduced to 0.03^2 to have at least 999 of 1000 conform to specification.

2-39. The life of an automotive battery is normally distributed with a mean 900 days and standard deviation 35 day. What fraction of these batteries would be expected to survive beyond 1000 days?

$x \sim N(900, 35^2)$

$$\Pr\{x > 1000\} = 1 - \Pr\{x \leq 1000\} = 1 - \Pr\left\{x \leq \frac{1000-900}{35}\right\} = 1 - \Phi(2.8571) = 1 - 0.9979 = 0.0021$$

The percentage expected to survive more than 1000 days is 0.21%.

2-41. The specifications of an electronic component in a target-acquisition systems are that its life must be between 5000 and 10,000 h. The life is normally distributed with mean 7500 h. The manufacturer realizes a price of $10 per unit produced; however, defective units must be replaced at a cost of $5 to the manufacturer. Two different manufacturing processes can be used, both of which have the same mean life. However, the standard deviation of life for process 1 is 1000h, whereas for process 2 it is only 500 h. Production costs for process 2 are twice those for process 1. What value of production costs will determine the selection between process 1 and 2?

The information required for this exercise is:
$x_1 \sim N(7500, \sigma_1^2 = 1000^2)$; $x_2 \sim N(7500, \sigma_2^2 = 500^2)$; LSL=5,000 h; USL=10,000 h
sales = \$10/unit, defect = \$5/unit, profit = \$10 × Pr{good} + \$5 × Pr{bad} − c

For Process 1

$$\text{proportion defective} = p_1 = 1-\Pr\{\text{LSL} \le x_1 \le \text{USL}\} = 1-\Pr\{x_1 \le \text{USL}\} + \Pr\{x_1 \le \text{LSL}\}$$
$$= 1-\Pr\left\{z_1 \le \frac{10{,}000-7{,}500}{1{,}000}\right\} + \Pr\left\{z_1 \le \frac{5{,}000-7{,}500}{1{,}000}\right\}$$
$$= 1-\Phi(2.5)+\Phi(-2.5) = 1-0.9938+0.0062 = 0.0124$$

profit for process 1 = 10 (1 − 0.0124) + 5 (0.0124) − c_1 = 9.9380 − c_1

For Process 2

$$\text{proportion defective} = p_2 = 1-\Pr\{\text{LSL} \le x_2 \le \text{USL}\} = 1-\Pr\{x_2 \le \text{USL}\} + \Pr\{x_2 \le \text{LSL}\}$$
$$= 1-\Pr\left\{z_2 \le \frac{10{,}000-7{,}500}{500}\right\} + \Pr\left\{z_2 \le \frac{5{,}000-7{,}500}{500}\right\}$$
$$= 1-\Phi(5)+\Phi(-5) = 1-1.0000+0.0000 = 0.0000$$

profit for process 2 = 10 (1 − 0.0000) + 5 (0.0000) − c_2 = 10 − c_2

If $c_2 > c_1 + 0.0620$, then choose process 1

CHAPTER 3

Inferences about Process Quality

CHAPTER GOALS

After completing this chapter, you will be able to:

1. Sample from Normal, Bernoulli, Poisson distributions
2. Understand and compute point estimates from data sets
3. Make decisions on process data by using statistical inference:
   - Define null and alternate hypothesis
   - Choose and compute test statistics
   - Define critical or reject regions
   - Compare test statistics to critical or reject regions
   - Estimate confidence intervals
4. Generate normal probability plots of data
5. Compare two samples of data
6. Compare multiple samples with analysis of variance

Exercises

3-1. The inside diameter of bearings used in an aircraft landing gear assembly are known to have a standard deviation of $\sigma = 0.002$ cm. A random sample of 15 bearings has an average inside diameter of 8.2535 cm.

(a) Test the hypothesis that the mean inside bearing diameter is 8.25 cm. Use a two-side alternative and $\alpha = 0.05$.

$n = 15$; $\bar{x} = 8.2535$ cm; $\sigma = 0.002$ cm

$\mu_0 = 8.25$, $\alpha = 0.05$

Test: H_0: $\mu = 8.25$ vs. H_1: $\mu \neq 8.25$. Reject H_0 if $|Z_0| > Z_{\alpha/2}$.

First calculate the test statistic Z_0 by

$$Z_0 = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{8.2535 - 8.25}{0.002/\sqrt{15}} = 6.78$$

From Appendix Table II, identify

$$Z_{\alpha/2} = Z_{0.05/2} = Z_{0.025} = 1.96$$

Reject H_0: $\mu = 8.25$, and conclude that the mean bearing ID is not equal to 8.25 cm.

(b) Find the P-value for this test.

From Appendix Table II, find the corresponding cumulative Standard Normal, $\Phi(Z_0)$ for $Z_0 = 6.78$ and calculate:

$$P = 2[1 - \Phi(Z_0)] = 2[1 - \Phi(6.78)] = 2[1 - 1.00000] = 0$$

(c) Construct a 95% two-sided confidence interval on mean bearing diameter.

By using equation 3-29, the confidence interval on the mean bearing diameter is calculated as follows:

$$\bar{x} - Z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \le \mu \le \bar{x} + Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

$$8.2535 - 1.96\left(\frac{0.002}{\sqrt{15}}\right) \le \mu \le 8.2535 + 1.96\left(\frac{0.002}{\sqrt{15}}\right)$$

$$8.2525 \le \mu \le 8.2545$$

3-3. The life of a battery used in a cardiac pacemaker is assumed to be normally distributed. A random sample of 10 batteries is subjected to an accelerated life test by running them continuously at an elevated temperature until failure, and the following lives are obtained.

25.5 h	26.1 h
26.8	23.2
24.2	28.4
25.0	27.8
27.3	25.7

(a) The manufacturer wants to be certain that the mean battery life exceeds 25 h. What conclusions can be drawn from these data (use $\alpha = 0.05$)?

Because σ is unknown, and estimate, S, is calculated and the t-distribution used, one-sided, as follows:

$x \sim N(\mu, \sigma)$, $\bar{x} = 26.0$, $S = 1.62$, $\mu_0 = 25$, $\alpha = 0.05$

Test H_0: $\mu = 25$ vs. H_1: $\mu > 25$. Reject H_0 if $t_0 > t_\alpha$.

$$t_0 = \frac{\bar{x} - \mu_0}{S/\sqrt{n}} = \frac{26.0 - 25}{1.62/\sqrt{10}} = 1.952$$

$$t_{\alpha,n-1} = t_{0.05,10-1} = 1.833$$

Reject H_0: $\mu = 25$, and conclude that the mean life exceeds 25 h.

These calculations are validated with Minitab as follows:

```
Test of mu = 25 vs mu > 25
Variable          N      Mean     StDev   SE Mean
Ex3-3            10    26.000     1.625     0.514

Variable       95.0% Lower Bound        T       P
Ex3-3                     25.058     1.95   0.042
```

(b) Construct a 90% two-sided confidence interval on mean life in the accelerated test.

With $\alpha = 0.10$, and equation 3-34, the confidence interval on mean life in the accelerated test is:

$$\bar{x} - t_{\alpha/2,n-1}\frac{S}{\sqrt{n}} \le \mu \le \bar{x} + t_{\alpha/2,n-1}\frac{S}{\sqrt{n}}$$

$$26.0 - 1.833\left(\frac{1.62}{\sqrt{10}}\right) \le \mu \le 26.0 + 1.833\left(\frac{1.62}{\sqrt{10}}\right)$$

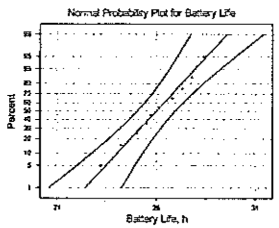
$$25.06 \le \mu \le 26.94$$

Again, Minitab validates the calculations as follows:

```
Variable          N      Mean     StDev   SE Mean          90.0% CI
Ex3-3            10    26.000     1.625.    0.514   (  25.058,   26.942)
```

(c) Construct a normal probability plot of the battery life data. What conclusions can you draw?

In Minitab, select "Options," then "Obtain Plot Points Using - Modified Kaplan-Meier Method." Using Minitab, the normal probability plot is:



The plotted points fall approximately along a straight line, so the assumption that battery life is normally distributed is appropriate.

3-5. A new process has been developed for applying photoresist to 125-mm silicon wafers used in manufacturing integrated circuits. Ten wafers were tested, and the photoresist thickness measurements shown here were observed:

13.3946 (x 1000 angstroms)	13.4002 (x 1000 angstroms)
13.3987	13.3957
13.3902	13.4015
13.4001	13.3918
13.3965	13.3925

(a) Test the hypothesis that mean thickness is 13.4 x 1000 Å. Use $\alpha = 0.05$ and assume a two-sided alternative.

First, estimate the mean and standard deviation: $x \sim N(\mu, \sigma)$, $n = 10$, $\bar{x} = 13.39618 (\times 1000\ \text{Å})$, $S = 0.00391$

$\mu_0 = 13.4 \times 1000$ Å, $\alpha = 0.05$

Test: H_0: $\mu = 13.4$ vs. H_1: $\mu \neq 13.4$. Reject H_0 if $|t_0| > t_{\alpha/2}$.

Compute the test statistic as follows:

$$t_0 = \frac{\bar{x} - \mu_0}{S/\sqrt{n}} = \frac{13.39618 - 13.4}{0.00391/\sqrt{10}} = -3.089$$

Identify $t_{\alpha/2,n-1}$ from Appendix Table IV: $t_{\alpha/2,n-1} = t_{0.025,9} = 2.262$

Reject H_0: $\mu = 13.4$, and conclude that the mean thickness differs from 13.4×1000 Å.

This test can also be performed in Minitab as follows:

```
Test of mu = 13.4 vs mu not = 13.4
Variable          N      Mean     StDev   SE Mean
Ex3-5            10   13.3962    0.0039    0.0012
Variable              95.0% CI            T       P
Ex3-5          ( 13.3934, 13.3990)    -3.09   0.013
```

(b) Find a 99% tow-sided confidence interval on mean photoresist thickness. Assume that the thickness is normally distributed.

With $\alpha = 0.01$, identify $t_{\alpha/2,n-1}$ from Appendix Table IV and calculate the confidence interval on mean photoresist thickness as:

$$\bar{x} - t_{\alpha/2,n-1}\frac{S}{\sqrt{n}} \leq \mu \leq \bar{x} + t_{\alpha/2,n-1}\frac{S}{\sqrt{n}}$$

$$13.39618 - 3.2498\left(\frac{0.00391}{\sqrt{10}}\right) \leq \mu \leq 13.39618 + 3.2498\left(\frac{0.00391}{\sqrt{10}}\right)$$

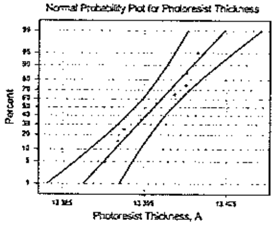
$$13.39216 \leq \mu \leq 13.40020$$

Using Minitab, the confidence interval calculations are verified:

```
Variable          N      Mean     StDev   SE Mean          99.0% CI
Ex3-5            10   13.3962    0.0039    0.0012   ( 13.3922, 13.4002)
```

(c) Does the normality assumption seem reasonable for these data?

Note: In Minitab, select Options, then "Obtain Plot Points Using - Modified Kaplan-Meier Method"



The plotted points form a reverse "S" shape, instead of a straight line, so the assumption that battery life is normally distributed is not appropriate.

3-7. Ferric chloride is used as a flux in some types of extraction metallurgy processes. This material is shipped in containers, and the container weight varies. It is important to obtain an accurate estimate of mean container weight. Suppose that from long experience a reliable value for the standard deviation of flux container weight is determined to be 4 lb. How large a sample would be required to construct a 95% two-sided confidence interval on the mean that has a total width of 1 lb?

$\sigma = 4$ lb, $\alpha = 0.05$, $Z_{\alpha/2} = Z_{0.025} = 1.9600$, total confidence interval width = 1 lb, find n

$$2\left[Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right] = \text{total width}$$

$$2\left[1.9600\frac{4}{\sqrt{n}}\right] = 1$$

$$n = 246$$

A sample size of 246 would be required.

3-9. The output voltage of a power supply is assumed to be normally distributed. Sixteen observations taken at random on voltage are shown here.

10.35	9.30	10.00	9.96
11.65	12.00	11.25	9.58
11.54	9.95	10.28	8.37
10.44	9.25	9.38	10.85

$x \sim N(\mu, \sigma)$, $n = 16$; $\bar{x} = 10.259$ V; $S = 0.999$ V

(a) Test the hypothesis that the mean voltage equals 12 V against a two-sided alternative using $\alpha = 0.05$.

$\mu_0 = 12$, $\alpha = 0.05$
Test H_0: $\mu = 12$ vs. H_1: $\mu \neq 12$. Reject H_0 if $|t_0| > t_{\alpha/2}$.

$$t_0 = \frac{\bar{x} - \mu_0}{S/\sqrt{n}} = \frac{10.259 - 12}{0.999/\sqrt{16}} = -6.971$$

$t_{\alpha/2,n-1} = t_{0.025,16-1} = 2.131$
Reject H_0: $\mu = 12$, and conclude that the mean output voltage differs from 12V.

The Minitab output for this exercise is shown below. The t value calculated by Minitab agrees with the calculations above. Also note that the hypothesis test value of 12 is not included in the 95% confidence interval of 9.727 to 10.792.

```
Test of mu = 12 vs mu not = 12
Variable              N       Mean      StDev    SE Mean
Ex3-9                16     10.259      0.999      0.250
Variable                 95.0% CI              T        P
Ex3-9             (   9.727,   10.792)     -6.97    0.000
```

(b) Construct a 95% two-sided confidence interval on μ.

With $\alpha = 0.05$, $t_{\alpha/2,n-1} = t_{0.025,16-1} = 2.131$, the confidence interval is calculated as:

$$\bar{x} - t_{\alpha/2,n-1}\frac{S}{\sqrt{n}} \leq \mu \leq \bar{x} + t_{\alpha/2,n-1}\frac{S}{\sqrt{n}}$$

$$10.259 - 2.131\left(\frac{0.999}{\sqrt{16}}\right) \leq \mu \leq 10.259 + 2.131\left(\frac{0.999}{\sqrt{16}}\right)$$

$$9.727 \leq \mu \leq 10.791$$

The Minitab output for this exercise is:

```
Variable             N       Mean      StDev    SE Mean          95.0% CI
Ex3-9               16     10.259      0.999      0.250   (   9.727,   10.792)
```

(c) Test the hypothesis that $\sigma^2 = 1$ using $\alpha = 0.05$.

$\sigma^2_0 = 1$, $\alpha = 0.05$
Test H_0: $\sigma^2 = 1$ vs. H_1: $\sigma^2 \neq 1$. Reject H_0 if $\chi^2_0 > \chi^2_{\alpha/2,n-1}$ or $\chi^2_0 < \chi^2_{1-\alpha/2,n-1}$.

$$\chi_0^2 = \frac{(n-1)S^2}{\sigma_0^2} = \frac{(16-1)0.999^2}{1} = 14.970$$

$$\chi^2_{\alpha/2,n-1} = \chi^2_{0.025,16-1} = 27.488$$

$$\chi^2_{1-\alpha/2,n-1} = \chi^2_{0.975,16-1} = 6.262$$

Do not reject H_0: $\sigma^2 = 1$, and conclude that there is insufficient evidence that the variance differs from 1.

(d) Construct a 95% two-sided confidence interval on σ.

$\alpha = 0.05$; $\chi^2_{\alpha/2,n-1} = \chi^2_{0.025,15} = 27.488$; $\chi^2_{1-\alpha/2,n-1} = \chi^2_{0.975,15} = 6.262$

$$\frac{(n-1)S^2}{\chi^2_{\alpha/2,n-1}} \leq \sigma^2 \leq \frac{(n-1)S^2}{\chi^2_{1-\alpha/2,n-1}}$$

$$\frac{(16-1)0.999^2}{27.488} \leq \sigma^2 \leq \frac{(16-1)0.999^2}{6.262}$$

$$0.545 \leq \sigma^2 \leq 2.391$$

$$0.738 \leq \sigma \leq 1.546$$

(e) Construct a 95% upper confidence interval on σ.

$\alpha = 0.05$; $\chi^2_{1-\alpha,n-1} = \chi^2_{0.95,15} = 7.2609$

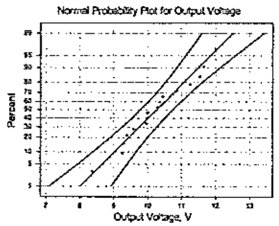
$$\sigma^2 \leq \frac{(n-1)S^2}{\chi^2_{1-\alpha,n-1}}$$

$$\sigma^2 \leq \frac{(16-1)0.999^2}{7.2609}$$

$$\sigma^2 \leq 2.062$$

$$\sigma \leq 1.436$$

(f) Does the assumption of normality seem reasonable for the output voltage?



From visual examination of the plot, the assumption of a normal distribution for output voltage seems appropriate.

3-11. Two quality control technicians measured the surface finish of a metal part, obtaining the data shown. Assume that the measurements are normally distributed.

Technician 1	Technician 2
1.45	1.54
1.37	1.41
1.21	1.56
1.54	1.37
1.48	1.20
1.29	1.31
1.34	1.27
	1.35

(a) Test the hypothesis that the mean surface finish measurements made by the two technicians are equal. Use $\alpha = 0.05$ and assume equal variances.

```
Two-sample T for Ex3-11T1 vs Ex3-11T2
           N      Mean      StDev   SE Mean
Ex3-11T1   7     1.383      0.115     0.043
Ex3-11T2   8     1.376      0.125     0.044
Difference = mu Ex3-11T1 - mu Ex3-11T2
Estimate for difference:   0.0066
95% CI for difference:  (-0.1283, 0.1415)
T-Test of difference = 0 (vs not =): T-Value = 0.11  P-Value = 0.917  DF = 12
```

Do not reject H_0: $\mu_1 - \mu_2 = 0$, and conclude that there is not sufficient evidence of a difference between measurements obtained by the two technicians.

(b) What are the practical implications of the test in part (a)? Discuss what practical conclusions you would draw if the null hypothesis were rejected?

The practical implication of this test is that it does not matter which technician measures parts; the readings will be the same. If the null hypothesis had been rejected, we would have been concerned that the technicians obtained different measurements, and an investigation should be undertaken to understand why.

(c) Assuming that the variances are equal, construct a 95% confidence interval on the mean difference in the surface-finish measurements.

$n_1 = 7;\ \bar{x}_1 = 1.383;\ S_1 = 0.115;\ n_2 = 8;\ \bar{x}_2 = 1.376;\ S_2 = 0.125$

$\alpha = 0.05; t_{\alpha/2, n_1+n_2-2} = t_{0.025,13} = 2.1604$

Calculate the pooled standard deviation:

$$S_p = \sqrt{\frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}} = \sqrt{\frac{(7-1)0.115^2 + (8-1)0.125^2}{7+8-2}} = 0.120$$

$$(\bar{x}_1 - \bar{x}_2) - t_{\alpha/2, n_1+n_2-2} S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \le (\mu_1 - \mu_2) \le (\bar{x}_1 - \bar{x}_2) + t_{\alpha/2, n_1+n_2-2} S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$(1.383 - 1.376) - 2.1604\sqrt{\frac{1}{7} + \frac{1}{8}} \le (\mu_1 - \mu_2) \le (1.383 - 1.376) + 2.1604\sqrt{\frac{1}{7} + \frac{1}{8}}$$

$$-0.127 \le (\mu_1 - \mu_2) \le 0.141$$

The confidence interval on the mean difference includes zero; therefore, there is no difference in the means. The Minitab output for this data is:

```
Two-sample T for Ex3-11T1 vs Ex3-11T2
           N      Mean      StDev   SE Mean
Ex3-11T1   7     1.383      0.115     0.043
Ex3-11T2   8     1.376      0.125     0.044
Difference = mu Ex3-11T1 - mu Ex3-11T2
Estimate for difference:   0.0066
95% CI for difference:  (-0.1280, 0.1412)
T-Test of difference = 0 (vs not =): T-Value = 0.11  P-Value = 0.917  DF = 13
Both use Pooled StDev = 0.120
```

The confidence interval for the difference contains zero. We can conclude that there is no difference in measurements obtained by the two technicians.

(d) Test the hypothesis that the variances of the measurements made by the two technicians are equal. Use $\alpha = 0.05$. What are the practical implications if the null hypothesis is rejected?

$\alpha = 0.05$

Test $H_0: \sigma_1^2 = \sigma_2^2$ versus $H_1: \sigma_1^2 \ne \sigma_2^2$. Reject H_0 if $F_0 > F_{\alpha/2, n_1-1, n_2-1}$ or $F_0 < F_{1-\alpha/2, n_1-1, n_2-1}$.

$$F_0 = \frac{S_1^2}{S_2^2} = \frac{0.115^2}{0.125^2} = 0.8464$$

$$F_{\alpha/2, n_1-1, n_2-1} = F_{0.05/2, 7-1, 8-1} = F_{0.025,6,7} = 5.119$$

$$F_{1-\alpha/2, n_1-1, n_2-1} = F_{1-0.05/2, 7-1, 8-1} = F_{0.975,6,7} = 0.176$$

```
Level1      Ex3-11T1
Level2      Ex3-11T2
ConfLvl     95.0000

F-Test (normal distribution)
Test Statistic: 0.846
P-Value       : 0.854
```

Do not reject H_0, and conclude that there is no difference in variability of measurements obtained by the two technicians. If the null hypothesis is rejected, we would have been concerned about the difference in measurement variability between the technicians, and an investigation should be undertaken to understand why.

(e) Construct a 95% confidence interval estimate of the ratio of the variances of technician measurement error.

$\alpha = 0.05;\ F_{1-\alpha/2,n_2-1,n_1-1} = F_{0.975,7,6} = 0.1954;\ F_{\alpha/2,n_2-1,n_1-1} = F_{0.025,7,6} = 5.6955$

$$\frac{S_1^2}{S_2^2} F_{1-\alpha/2,n_2-1,n_1-1} \le \frac{\sigma_1^2}{\sigma_2^2} \le \frac{S_1^2}{S_2^2} F_{\alpha/2,n_2-1,n_1-1}$$

$$\frac{0.115^2}{0.125^2}(0.1954) \le \frac{\sigma_1^2}{\sigma_2^2} \le \frac{0.115^2}{0.125^2}(5.6955)$$

$$0.165 \le \frac{\sigma_1^2}{\sigma_2^2} \le 4.821$$

The confidence interval includes one; therefore, there is not difference in the variances.

(f) Construct a 95% confidence interval on the variance of measurement error for technician 2.

$n_2 = 8;\ \bar{x}_2 = 1.376;\ S_2 = 0.125$

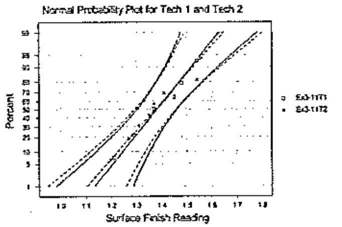
$\alpha = 0.05;\ \chi^2_{\alpha/2,n_2-1} = \chi^2_{0.025,7} = 16.0128;\ \chi^2_{1-\alpha/2,n_2-1} = \chi^2_{0.975,7} = 1.6899$

$$\frac{(n-1)S^2}{\chi^2_{\alpha/2,n-1}} \le \sigma^2 \le \frac{(n-1)S^2}{\chi^2_{1-\alpha/2,n-1}}$$

$$\frac{(8-1)0.125^2}{16.0128} \le \sigma^2 \le \frac{(8-1)0.125^2}{1.6899}$$

$$0.007 \le \sigma^2 \le 0.065$$

(g) Does the normality assumption seem reasonable for the data?



The normality assumption seems reasonable for these readings.

3-13. Two different hardening processes, (1) saltwater quenching and (2) oil quenching, are used on samples of a particular type of metal alloy. The results are shown here. Assume that hardness is normally distributed.

Saltwater Quench	Oil Quench
145	152
150	150
153	147
148	155
141	140
152	146
146	158
154	152
139	151
148	143

Saltwater quench: $n_1 = 10;\ \bar{x}_1 = 147.6;\ S_1 = 4.97$

Oil quench: $n_2 = 10;\ \bar{x}_2 = 149.4;\ S_2 = 5.46$

(a) Test the hypothesis that the mean hardness for the saltwater quenching process equals the mean hardness for the oil quenching process. Use $\alpha = 0.05$ and assume equal variances.

Assume $\sigma_1^2 = \sigma_2^2$

```
Two-sample T for Ex3-13SQ vs Ex3-13OQ
           N      Mean     StDev   SE Mean
Ex3-13SQ  10    147.60      4.97       1.6
Ex3-13OQ  10    149.40      5.46       1.7
Difference = mu Ex3-13SQ - mu Ex3-13OQ
Estimate for difference:  -1.80
95% CI for difference: (-6.73, 3.13)
T-Test of difference = 0 (vs not =): T-Value = -0.77  P-Value = 0.451  DF = 17
```

Do not reject H_0, and conclude that there is no difference between the quenching processes.

(b) Assuming that the variances σ_1^2 and σ_2^2 are equal, construct a 95% confidence interval on the difference in mean hardness.

From the Minitab output in part (a), $S_1 = 4.97$ and $S_2 = 5.46$.

$\alpha = 0.05;\ t_{\alpha/2,n_1+n_2-2} = t_{0.025,18} = 2.1009$

The pooled standard deviation is

$$S_p = \sqrt{\frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}} = \sqrt{\frac{(10-1)4.97^2 + (10-1)5.46^2}{10+10-2}} = 5.22$$

$$(\bar{x}_1-\bar{x}_2)-t_{\alpha/2,n_1+n_2-2}S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}} \le (\mu_1-\mu_2) \le (\bar{x}_1-\bar{x}_2)+t_{\alpha/2,n_1+n_2-2}S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$$

$$(147.6-149.4)-2.1009\sqrt{\frac{1}{10}+\frac{1}{10}} \le (\mu_1-\mu_2) \le (147.6-149.4)+2.1009\sqrt{\frac{1}{10}+\frac{1}{10}}$$

$$-6.7 \le (\mu_1-\mu_2) \le 3.1$$

Because the confidence interval of the difference in means includes zero, there is no difference in means.

(c) Construct a 95% confidence interval on the ratio σ_1^2/σ_2^2. Does the assumption made earlier of equal variances seem reasonable?

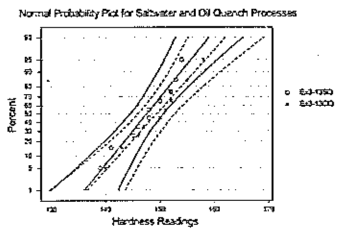
$\alpha=0.05$; $F_{1-\alpha/2,n_2-1,n_1-1}=F_{0.975,9,9}=0.2484$; $F_{\alpha/2,n_2-1,n_1-1}=F_{0.025,9,9}=4.0260$

$$\frac{S_1^2}{S_2^2}F_{1-\alpha/2,n_2-1,n_1-1} \le \frac{\sigma_1^2}{\sigma_2^2} \le \frac{S_1^2}{S_2^2}F_{\alpha/2,n_2-1,n_1-1}$$

$$\frac{4.97^2}{5.46^2}(0.2484) \le \frac{\sigma_1^2}{\sigma_2^2} \le \frac{4.97^2}{5.46^2}(4.0260)$$

$$0.21 \le \frac{\sigma_1^2}{\sigma_2^2} \le 3.34$$

(d) Does the assumption of normality seem appropriate for these data?



The normal distribution assumptions for both the saltwater and oil quench methods seem reasonable. However, the slopes on the normal probability plots do not appear to be the same, so the equal variance assumptions do not seem reasonable.

3-15. A random sample of 500 connecting rod pins contains 65 nonconforming units. Estimate the process fraction nonconforming.

(a) Test the hypothesis that the true fraction defective in this process is 0.08. Use $\alpha=0.05$.

$n=500$; $x=65$; $\hat{p}=x/n=65/500=0.130$

$p_0=0.08$, $\alpha=0.05$. Test H_0: $p=0.08$ versus H_1: $p\neq 0.08$. Reject H_0 if $|Z_0|>Z_{\alpha/2}$.

$np_0=500(0.08)=40$

Since $(x=65)>(np_0=40)$,

$$Z_0=\frac{(x-0.5)-np_0}{\sqrt{np_0(1-p_0)}}=\frac{(65-0.5)-40}{\sqrt{40(1-0.08)}}=4.0387$$

$Z_{\alpha/2}=Z_{0.05/2}=Z_{0.025}=1.96$

Reject H_0, and conclude that the sample process fraction nonconforming does differ from 0.08.

(b) Find the P-value for this test.

$$P=2\left[1-\Phi|Z_0|\right]=2\left[1-\Phi|4.0387|\right]=2[1-0.99997]=0.00006$$

(c) Construct a 95% upper confidence interval on the true process fraction nonconforming.

$\alpha=0.05$; $Z_\alpha=Z_{0.05}=1.645$

$$p \le \hat{p}+Z_\alpha\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$p \le 0.13+1.645\sqrt{\frac{0.13(1-0.13)}{500}}$$

$$p \le 0.155$$

The 95% upper confidence interval for the process fraction nonconforming is 0.155.

3-17. A new purification unit is installed in a chemical process. Before its installation, a random sample yielded the following data about the percentage of impurity: $\bar{x}_1=9.85$, $S_1^2=81.73$ and $n_1=10$. After installation, a random sample resulted in $\bar{x}_2=8.08$, $S_2^2=78.46$ and $n_2=8$.

before: $n_1=10$; $\bar{x}_1=9.85$; $S_1^2=81.73$

after: $n_2=8$; $\bar{x}_2=8.08$; $S_2^2=78.46$

(a) Can you conclude that the two variances are equal? Use $\alpha=0.05$.

Test $H_0:\sigma_1^2=\sigma_2^2$ versus $H_1:\sigma_1^2\neq\sigma_2^2$, at $\alpha=0.05$

Reject H_0 if $F_0>F_{\alpha/2,n_1-1,n_2-1}$ or $F_0<F_{1-\alpha/2,n_1-1,n_1-1}$

$F_{\alpha/2,n_1-1,n_2-1}=F_{0.025,9,7}=4.8232$; $F_{1-\alpha/2,n_1-1,n_1-1}=F_{0.975,9,7}=0.2383$

$$F_0=\frac{S_1^2}{S_2^2}=\frac{81.73}{78.46}=1.0417$$

$F_0=1.0417<4.8232$ and >0.2383, so do not reject H_0

The impurity variances before and after are the same.